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Table of Contents.

On the Intensities of $\lambda 4686$ and H_β in the Wolf-Rayet Stars.

In a previous paper (Poulk. Obs. Circ. № 4, p. 8) the writer has proposed a method for the determination of the temperatures of the nuclei of planetary nebulae from the intensity ratio $\frac{\lambda 4686}{H_\beta}$. This method gives the possibility of the determination of the temperatures of the faintest nuclei because it is not connected with the observations of the nuclei itself. However, this method is based on the assumption of the complete absorption of the radiation beyond the limit of Lymann series by the normal hydrogen atoms of nebula, as well as upon the assumption of complete absorption of radiation beyond the wave-length 230 Å by once ionized atoms of helium.

The first of these assumptions is certainly not fulfilled in the gaseous envelopes of the Wolf-Rayet stars and this is the reason why, though the mechanism of the production of the emission bands in Wolf-Rayet spectra is the same as of the nebular emission, our method cannot be applied to the Wolf-Rayet stars.

We have indeed that the temperatures of some Wolf-Rayet stars are of the order of 70 000° (V. Ambarzumian, Nature, *129*, 725, 1932; C. S. Beals, M. N. *92*, 677, 1932). For such a temperature we find for the intensity ratio $\frac{\lambda 4686}{H_\beta}$ the value $\frac{1}{100}$ (Poulk. Obs. Circ., № 4, p. 10 Table 1), if the assumptions upon which our method is based are fulfilled. However, a glance at the microphotograms of the Wolf-Rayet spectra convinces that H_β is fainter than $\lambda 4686$ in spite of the fact that H_β is blended with a line of *He* II. Therefore we shall assume that only a small part of the radiation between 230 Å and 918 Å is absorbed by hydrogen atoms. In such cases the application of our method will give absurdly high temperatures.

However, there is another important physical characteristic for the gaseous envelopes, which may be deduced from the intensity ratio $\frac{\lambda 4686}{H_\beta}$ independently of the fundamental assumptions of our previous paper. This characteristic will be discussed in the present note.

The total number of captures per second per c. c. of electrons on the nth level of hydrogen atoms is (Cilliè, M. N. *82*, 823, 1932)

$$n_p y \frac{2^9 \pi^5}{(6\pi)^{3/2}} \frac{\varepsilon^{10}}{m^2 c^3 h^3} \left(\frac{m}{kT}\right)^{3/2} \frac{1}{n^3} e^{\chi_n/kT} Ei\left(\frac{\chi_n}{kT}\right) \tag{1}$$

where n_p and y are the numbers of free protons and free electrons par c. c. respectively, ε is the charge of a proton, m — the mass of an electron, c, h and k are usual world-constants, T the temperature of the gas under consideration and χ_n — the ionization potential of hydrogen atom in the nth level. The expression (1) can be written in the form:

$$C n_p y M(n, T) \tag{2}$$

where

$$M(n, T) = \frac{1}{T^{3/2}} \frac{1}{n^3} e^{\chi_n/kT} Ei\left(\frac{\chi_n}{kT}\right). \tag{3}$$

Similarly, the total number of captures per second per c. c. of electrons on n th level of a hydrogenlike atom is

$$C n_\alpha y M'(n, T) \tag{4}$$

where

$$M'(n, T) = \frac{Z^4}{T^{3/2}} \frac{1}{n^3} e^{\chi_n'/kT} Ei\left(\frac{\chi_n'}{Tk}\right) \tag{5}$$

C is the same constant as in (2), n_α is the number of free nuclei with the charge $Z\varepsilon$ in c. c. and χ_n' is the nth ionization potential of the hydrogenlike atom with such nucleus. In case when $Z = 2$, n_α is the number of α particles or He^{++} atoms, and χ_n' is the nth ionization potential of He^+.

Let N_n be the number of hydrogen atoms per c. c. in the nth state at any moment. According to Cilliè we have

$$N_n \sum_{r=2}^{n-1} A_{n,r} = n_p y C M(n, T) + \sum_{r=1}^{\infty} N_{n+r} A_{n+r,n} \tag{6}$$

where $A_{s,r}$ is the probability of spontaneous transition $s \to r$ $(s > r)$.

Solving these equations we find for N_4

$$N_4 = 10^{-8} n_p y C[5.719\, M(4, T) + 2.087\, M(5, T) + \\ + 1.808\, M(6, T) + 1.860\, M(7, T) + \ldots] \tag{7}$$

For He^+ atoms the probabilities of spontaneous transitions are 16 time larger than the corresponding probabilities for hydrogen, and the equations for the numbers of He^+ atoms in the nth level N_n' ten may be written in the form

$$16 N_n' \sum_{r=2}^{n-1} A_{n,r} = n_\alpha y C M'(n, T) + 16 \sum_{r=1}^{\infty} N'_{n+r} A_{n+r,n} \tag{7}$$

Solving these equations we obtain

$$N_4' = 10^{-8} n_\alpha y \frac{C}{16} [5.719\, M(4, T) + 2.087\, M(5, T) + \\ + 1.808\, M(6, T) + 1.860\, M(7, T) + \ldots] \tag{8}$$

For the ratio $\frac{N_4'}{N_4}$ we find therefore

$$\frac{N_4'}{N_4} = \frac{n_\alpha}{n_p} \Theta$$

where

$$\Theta = \frac{1}{16} \frac{5.719 M'(4, T) + 2.087 M'(5, T) + 1.808 M'(6, T) + 1.860 M'(7, T) + \ldots}{5.719 M'(4, T) + 2.087 M(5, T) + 1.808 M(6, T) + 1.860 M(7, T) + \ldots} \tag{8'}$$

According to (3) and (5) we have

$$\frac{M'(n, T)}{M(n, T)} = Z^4 \frac{e^{\chi_n'/kT} Ei\left(\frac{\chi_n'}{kT}\right)}{e^{\chi_n'/kT} Ei\left(\frac{\chi_n}{kT}\right)}. \tag{9}$$

Now $\chi_n' = 4\chi_n$ and if we denote $\frac{\chi_n}{kT} = x$ we obtain

$$\frac{M'(n, T)}{M(n, T)} = Z^4 \frac{e^{4x} Ei(4x)}{e^x Eix} = Z^4 \frac{e^{4x} \int_{4x}^{\infty} \frac{e^{-t}}{t} dt}{e^x \int_{x}^{\infty} \frac{e^{-t}}{t} dt} = Z^4 \frac{\int_0^\infty \frac{e^{-y}}{y+4x} dy}{\int_0^\infty \frac{e^{-y}}{y+x} dy} = Z^4 Q \tag{10}$$

where

$$\frac{1}{4} < Q \leq 1 \tag{11}$$

for

$$\frac{1}{4} \int_0^\infty \frac{e^{-y}}{y+x} dx < \int_0^\infty \frac{e^{-y}}{y+4x} dy < \int_0^\infty \frac{e^{-y}}{y+x} dy$$

Thus

$$4 < \frac{M'(n, T)}{M(n, T)} < 16$$

and

$$\frac{1}{4} < \Theta < 1. \tag{12}$$

The intensity ratio $\frac{\lambda 4686}{H\beta} = \frac{I'_{43}}{I_{42}}$ is connected with N_4 and N_4' by means of equation

$$\frac{I'_{43}}{I_{42}} = \frac{A'_{43}}{A_{42}} \frac{h\nu'_{43}}{h\nu'_{42}} \frac{N_4'}{N_4}. \tag{13}$$

Comparing (8) and (13) we find

$$\frac{n_\alpha}{n_p} = \frac{1}{\Theta} \frac{\lambda'_{43}}{\lambda_{42}} \frac{A_{42}}{A'_{43}} \frac{I'_{43}}{I_{42}} \tag{14}$$

From the known values of A_{42}, A'_{43}, λ'_{43} and λ_{42} we have

$$\frac{\lambda'_{43}}{\lambda_{42}} \frac{A_{42}}{A'_{43}} = \frac{1}{18}.$$

Further from (12) we find

$$1 < \frac{1}{\Theta} < 4$$

Therefore

$$\frac{1}{18} \frac{I'_{43}}{I_{42}} < \frac{n_\alpha}{n_p} < \frac{2}{9} \frac{I'_{43}}{I_{42}} \tag{16}$$

In a private letter Dr. C. S. Beals has communicated to the writer some results of the measurements of the bands of H and He^+ in the Wolf-Rayet stars. According to Beals we have $\frac{I_{42}}{I'_{43}} = 0.03$. For the ratio under consideration we obtain:

$$1.8 < \frac{n_\alpha}{n_p} < 7.2 \tag{17}$$

This result is to some degree astonishing. The ionization of H in the envelopes of the Wolf-Rayet stars will be more advanced than the ionization of He^+. The ratio of the number of He^+ atoms to the number of neutral atoms will be therefore many times larger than the ratio $\frac{n_\alpha}{n_p}$. In addition there is a small number of neutral He atoms. It is clear that for the ratio of a total number of He atoms in all states of ionization $n_{\alpha+He^{+}+He}$ to the total number of hydrogen atoms (ionized and neutral) n_{p+H} we shall have

$$\frac{n_{\alpha+He^{+}+He}}{n_{p+H}} > 1.8$$

Thus helium is more abundant in the envelopes of the Wolf-Rayet stars than hydrogen. The problem whether this abundance of helium is connected with the selective radiation pressure, or with the composition of the outer layer of the Wolf-Rayet stars cannot be treated here.

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